Spin-$\frac{5}{2}$ fields in hadron physics. [*]

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We show that the Lagrangian of the free spin-$\frac{5}{2}$ field in the spinor-tensor representation with the auxiliary spinor field depends on the three arbitrary parameters. The first two parameters are associated with the spin-$\frac{3}{2}$ and -$\frac{1}{2}$ sector of the theory while the latter one is related to the auxiliary degrees of freedom. We derive a corresponding propagator of the system which represents (2 x 2) matrix in the $(\psi_{\mu\nu}, \xi)$ space. The diagonal terms stand for the propagation of the spin-$\frac{5}{2}$ and auxiliary fields whereas the non-diagonal ones correspond to the $\psi_{\mu\nu} - \xi$ mixing. The resulting spin-$\frac{5}{2}$ propagator contains non-pole contributions coming from the spin-$\frac{3}{2}$ and -$\frac{1}{2}$ sector of the spinor-tensor representation. A general form of the interaction vertex involving spin-$\frac{5}{2}$ field is discussed on the example of the $\pi N N_{\frac{5}{2}}^*$ coupling. It is demonstrated that lower spin degrees of freedom can be removed from the theory by using higher order derivative coupling.

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I. INTRODUCTION

The description of pion- and photon-induced reactions in the resonance energy region is faced with the problem of proper treatment of higher spin states. In 1941 Rarita and Schwinger (R-S) suggested a set of equations which a field function of a higher spin should obey [1]. Another formulation has been developed by Fierz and Pauli [2] where an auxiliary field concept is used to derive subsidiary constraints on the field function.

Regardless of the procedure used the obtained Lagrangians for free higher-spin fields turn out to be always dependent on arbitrary free parameters. For the spin-$\frac{3}{2}$ fields this issue is widely discussed in the Literature: (see e.g. [3, 4, 5] for a modern status of the problem). The case of the spin-$\frac{5}{2}$ fields is less studied. First attempts in this way have been made in [6, 7] where a theory of free fields has been suggested. The authors of [7] deduced an equation of motion as a decomposition in terms of corresponding projection operators with additional algebraic constraints on parameters of the decomposition. Schwinger [6] derived a particular form of the spin-$\frac{5}{2}$ equation which coincides with the equation suggested in [7] for a specific choice of the parameters.

The free particle propagator is a central quantity in most of the calculations in quantum field theory. In [7] the authors deduced a spin-$\frac{5}{2}$ propagator written in operator form. In practical calculations, however, one needs an explicit expression of the propagator. An attempt to construct a propagator only from the spin-$\frac{5}{2}$ projection operator has been made in [8]. We demonstrate that such a quantity is not consistent with the equation of motions for the spin-$\frac{5}{2}$ field. Another pathology is experienced with the propagator [9] and projector [10] used in calculations of the resonance production amplitudes: they do not fulfill the condition $[\gamma_0 G_{\mu\nu;\rho\sigma}^{\frac{5}{2}}]^\dagger = \gamma_0 G_{\rho\sigma;\mu\nu}^{\frac{5}{2}}$ and consequently are not hermitian. Therefore it is important to derive the propagator and investigate its properties in detail. To our knowledge no such a study has been done so far.

The aim of the paper is to deduce an explicit expression for the spin-$\frac{5}{2}$ propagator and study its properties. Guided by the properties of the free spin-$\frac{3}{2}$ R-S theory one would expect the equation of motion for the spin-$\frac{5}{2}$ field has two arbitrary free parameters which define the non-pole spin-$\frac{3}{2}$ and-$\frac{1}{2}$ contributions to the full propagator. The coupling of the spin-$\frac{5}{2}$ field to the (e.g.) pion-nucleon final state is therefore defined up to two 'off-shell' parameters [11] which scale the non-pole contributions to the physical observables. Hence, one can ask whether such an arbitrariness can be removed from the theory.

The possibility to construct consistent higher-spin massless theories has already been pointed out by Weinberg and Witten a while ago [12]. Pascalutsa has shown that by using a gauge invariant coupling for higher spin fields it is possible to remove the extra-degrees of freedom [13] in a particular case of the R-S theory which maintains gauge invariance in the massless limit.

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As we demonstrated in [4] the demand for the gauge-invariance may not be enough to eliminate the extra degrees of freedom at the interaction vertex. The problem appears when the theory does not have a massless limit. However, a coupling which removes non-pole terms from the spin-$\frac{5}{2}$ propagator can be easily constructed by using higher order derivatives. A corresponding interaction Lagragian has been deduced in [4] for the case of spin-$\frac{3}{2}$ fields and can be easily extended to higher spins too.

The paper is organized as follows: in Section II we suggest an alternative form of the free spin-$\frac{5}{2}$ Lagrangian as compared to [7] and discuss its properties in details. The presence of auxiliary field complicates the derivation of the propagator. Therefore, in Section III we first demonstrate how the free propagator can be obtained for a vector field in the presence of an auxiliary one. The method is then applied to the spin-$\frac{5}{2}$ field, Section IV. The resulting spin-$\frac{5}{2}$ propagator contains contributions corresponding to the lower spin-$\frac{3}{2}$,-$\frac{1}{2}$ sector of the spin-tensor representation. In Section V we discuss how these degrees of freedom can be removed from the physical observables the the example of the pion-nucleon scattering amplitude. The results are summarized in Section VI.

II.   FREE SPIN-$\frac{5}{2}$ FIELD.

The field function of higher spins in a spinor-tensor representation is a solution of the set of equations suggested by Rarita and Schwinger in [1]. In a consistent theory the description of the free field is specified by setting up an appropriate Lagrange function $\mathcal{L}(\psi_{\mu\nu}, \partial_\rho \psi_{\mu\nu})$. The spin-$\frac{5}{2}$ Lagrangian in the presence of the auxiliary spinor field $\xi(x)$ can be written in the form

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(\mathrm{aux})}, \tag{1}$$

where the explicit expressions for $\mathcal{L}^{(1)}$, $\mathcal{L}^{(2)}$, and $\mathcal{L}^{(\mathrm{aux})}$ read

$$
\begin{aligned}
\mathcal{L}^{(1)} = & \; \mathrm{i}\, a\, \bar{\psi}_{\mu\nu}(x)\Bigg((\gamma^\mu \mathrm{g}^{\nu\sigma} + \gamma^\nu \mathrm{g}^{\mu\sigma})\, \overrightarrow{\partial^\rho} + (\gamma^\mu \mathrm{g}^{\nu\rho} + \gamma^\nu \mathrm{g}^{\mu\rho})\, \overrightarrow{\partial^\sigma} \\
& \quad - (\gamma^\rho \mathrm{g}^{\nu\sigma} + \gamma^\sigma \mathrm{g}^{\nu\rho})\, \overleftarrow{\partial^\mu} - (\gamma^\rho \mathrm{g}^{\mu\sigma} + \gamma^\sigma \mathrm{g}^{\mu\rho})\, \overleftarrow{\partial^\nu} \Bigg) \psi_{\rho\sigma}(x) \\
& + \mathrm{i}\, \frac{F_1(a)}{2} \bar{\psi}_{\mu\nu}(x)\gamma^\lambda \left(\overrightarrow{\slashed{\partial}} - \overleftarrow{\slashed{\partial}} \right) \gamma^\delta \psi_{\rho\sigma}(x)\, \left(\mathrm{g}^{\lambda\mu}\mathrm{g}^{\delta\rho}\mathrm{g}^{\nu\sigma} + \mathrm{g}^{\lambda\nu}\mathrm{g}^{\delta\rho}\mathrm{g}^{\mu\sigma} + \mathrm{g}^{\lambda\mu}\mathrm{g}^{\delta\sigma}\mathrm{g}^{\nu\rho} + \mathrm{g}^{\lambda\nu}\mathrm{g}^{\delta\sigma}\mathrm{g}^{\mu\rho} \right) \\
& + m\, F_2(a)\, \bar{\psi}_{\mu\nu}(x)\, (\gamma^\mu \gamma^\rho \mathrm{g}^{\nu\sigma} + \gamma^\nu \gamma^\rho \mathrm{g}^{\mu\sigma} + \gamma^\mu \gamma^\sigma \mathrm{g}^{\nu\rho} + \gamma^\nu \gamma^\sigma \mathrm{g}^{\mu\rho})\, \psi_{\rho\sigma}(x) \\
& + \bar{\psi}_{\mu\nu}(x) \left(\frac{\mathrm{i}}{2} \left(\overrightarrow{\slashed{\partial}} - \overleftarrow{\slashed{\partial}} \right) - m \right) \psi_{\rho\sigma}(x)\, (\mathrm{g}^{\mu\rho}\mathrm{g}^{\nu\sigma} + \mathrm{g}^{\mu\sigma}\mathrm{g}^{\nu\rho}),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^{(2)} = & \; \mathrm{i}\, b\, \bar{\psi}_{\mu\nu}(x)\left(\mathrm{g}^{\mu\nu} \left(\gamma^\rho \overrightarrow{\partial^\sigma} + \gamma^\sigma \overrightarrow{\partial^\rho} \right) - \left(\gamma^\nu \overleftarrow{\partial^\mu} + \gamma^\mu \overleftarrow{\partial^\nu} \right) \mathrm{g}^{\rho\sigma} \right) \psi_{\rho\sigma}(x) \\
& + \mathrm{i}\, \frac{G_1(a,b)}{2}\, \bar{\psi}_{\mu\nu}(x)\mathrm{g}^{\mu\nu} \left(\overrightarrow{\slashed{\partial}} - \overleftarrow{\slashed{\partial}} \right) \mathrm{g}^{\rho\sigma} \psi_{\rho\sigma}(x) + m\, G_2(a,b)\, \bar{\psi}_{\mu\nu}(x)\mathrm{g}^{\mu\nu}\mathrm{g}^{\rho\sigma} \psi_{\rho\sigma}(x),
\end{aligned}
$$

$$
\mathcal{L}^{(\mathrm{aux})} = \; m\, c\, \left(\bar{\psi}_{\mu\nu}(x)\mathrm{g}^{\mu\nu}\xi(x) + \bar{\xi}(x)\mathrm{g}^{\rho\sigma}\psi_{\rho\sigma}(x) \right) + B(a,b,c)\, \bar{\xi}(x) \left(\frac{\mathrm{i}}{2}(\overrightarrow{\slashed{\partial}} - \overleftarrow{\slashed{\partial}}) + 3m \right) \xi(x), \tag{2}
$$

and $F_1(a)$, $F_2(a)$, $G_1(a,b)$, $G_2(a,b)$, and $B(a,b,c)$ are functions of the free real parameters a, b, and c, see Appendix B.

The Lagrangian eq. (1) in general depends on only three independent real parameters a, b, and c. This formulation of the spin-$\frac{5}{2}$ theory is simpler than that of suggested in [7]. In fact, the Lagrangian in [7] is written as a decomposition in terms of projection operators with a number of free parameters. These paramteres are subjected to additional subsiduary constraints need to be resolved.

Independent variations of $\psi_{\mu\nu}$ and ξ fields give two equations of motion which in momentum space can be written in the following form

$$\left(\Lambda^{(1)}_{\mu\nu;\rho\sigma}(p) + \Lambda^{(2)}_{\mu\nu;\rho\sigma}(p) \right) \psi^{\rho\sigma}(p) + c\, m\, \mathrm{g}^{\mu\nu}\xi(p) = 0, \tag{3}$$

$$m\, c\, \mathrm{g}^{\rho\sigma}\psi_{\rho\sigma}(p) + B(a,b,c)\, (\slashed{p} + 3m)\, \xi(p) = 0, \tag{4}$$

where the operators $\Lambda^{(1)}_{\mu\nu;\rho\sigma}(p)$, $\Lambda^{(2)}_{\mu\nu;\rho\sigma}(p)$ are

$$\Lambda^{(1)}_{\mu\nu;\rho\sigma}(p) = (\not{p} - m)(\mathrm{g}_{\mu\sigma}\mathrm{g}_{\nu\rho} + \mathrm{g}_{\mu\rho}\mathrm{g}_{\nu\sigma})$$
$$+ \; a(\gamma_\mu p_\rho \mathrm{g}_{\nu\sigma} + \gamma_\nu p_\rho \mathrm{g}_{\mu\sigma} + \gamma_\mu p_\sigma \mathrm{g}_{\nu\rho} + \gamma_\nu p_\sigma \mathrm{g}_{\mu\rho}$$
$$+ \quad \gamma_\rho p_\mu \mathrm{g}_{\nu\sigma} + \gamma_\sigma p_\mu \mathrm{g}_{\nu\rho} + \gamma_\rho p_\nu \mathrm{g}_{\mu\sigma} + \gamma_\sigma p_\nu \mathrm{g}_{\mu\rho})$$
$$+ \; F_1(a) \left(\gamma_\mu \not{p} \gamma_\rho \mathrm{g}_{\nu\sigma} + \gamma_\nu \not{p} \gamma_\rho \mathrm{g}_{\mu\sigma} + \gamma_\mu \not{p} \gamma_\sigma \mathrm{g}_{\nu\rho} + \gamma_\nu \not{p} \gamma_\sigma \mathrm{g}_{\mu\rho} \right)$$
$$+ \; m\, F_2(a) \left(\gamma_\mu \gamma_\rho \mathrm{g}_{\nu\sigma} + \gamma_\nu \gamma_\rho \mathrm{g}_{\mu\sigma} + \gamma_\mu \gamma_\sigma \mathrm{g}_{\nu\rho} + \gamma_\nu \gamma_\sigma \mathrm{g}_{\mu\rho} \right) \tag{5}$$

$$\Lambda^{(2)}_{\mu\nu;\rho\sigma}(p) = b\left(\gamma_\mu p_\nu \mathrm{g}_{\rho\sigma} + \gamma_\nu p_\mu \mathrm{g}_{\rho\sigma} + \gamma_\rho p_\sigma \mathrm{g}_{\mu\nu} + \gamma_\sigma p_\rho \mathrm{g}_{\mu\nu} \right)$$
$$+ \; \left(\not{p}\, G_1(a,b) + m\, G_2(a,b) \right) \mathrm{g}_{\mu\nu}\mathrm{g}_{\sigma\rho}. \tag{6}$$

The equations of motion $(3,4)$ are written in the most general form and are consistent with those of defined in [6, 7]. For example, the equation suggested by Schwinger corresponds to the choice of parameters $a = -1$, $b = 1$, and $c = -2$. Note that the functions $F_1(a)$, $F_2(a)$, $G_1(a,b)$, and $G_2(a,b)$ do not contain the parameter c which reflects independence of the spin-$\frac{5}{2}$ field on the axuliary degrees of freedom. The R-S constraints [1] follow from eqs. $(3,4)$ with the additional condition $\xi(p)=0$, see Appendix B.

It is interesting to note, that the operator $\Lambda^{(1)}_{\mu\nu;\rho\sigma}(p)$ would give an equation of motion $\Lambda^{(1)}_{\mu\nu;\rho\sigma}(p)\psi^{\mu\nu} = 0$ for the spin-$\frac{5}{2}$ fields provided $\mathrm{g}^{\mu\nu}\psi_{\mu\nu} = 0$, where the later property is assumed a priori. However, the corresponding inverse operator $[\Lambda^{(1)}_{\mu\nu;\rho\sigma}(p)]^{-1}$ has additional non-physical poles in the spin-$\frac{1}{2}$ sector. This indicates that the constraint $\mathrm{g}^{\mu\nu}\psi_{\mu\nu} = 0$ should also follow from the equation of motion and cannot be assumed a priori. The second operator $\Lambda^{(2)}_{\mu\nu;\rho\sigma}(p)$ acts only in the spin-$\frac{1}{2}$ sector of the spin-tensor representation. This can be checked by a direct decomposition of the operator (6) in terms of projection operators given in Appendix A . The same conclusion can be drawn from the observation that $\Lambda^{(2)}_{\mu\nu;\rho\sigma}(p)$ is orthogonal to all $\mathcal{P}^{\frac{5}{2}}_{\rho\sigma;\tau\delta}(p)$, $\mathcal{P}^{\frac{3}{2}}_{ij;\rho\sigma;\tau\delta}(p)$ projection operators, where $i,j = 1,2$. Hence the parameter b is related only to the spin-$\frac{1}{2}$ degrees of freedom whereas a scales both spin-$\frac{3}{2}$ and -$\frac{1}{2}$ ones.

In practical calculations one needs to know a free propagator corresponding to the spin-$\frac{5}{2}$ field. The derivation of the propagator becomes complicated in the presence of the auxiliary field degrees of freedom. To demonstrate the procedure it is useful to consider first an example of the free vector field φ_μ in the presence of auxiliary one. In the next Section we outline a general procedure which can be applied to the spin-$\frac{5}{2}$ case.

III.   FREE VECTOR FIELD

The idea to use auxiliary degrees of freedom to describe systems with higher spins was first utilized in the original work of Fierz and Pauli [2]. As is well known, however, there is no need for such complications in the case of spins $J \leq 2$ and $J \leq \frac{3}{2}$. For higher spins the use of axiliary degrees of becomes inevitable [7]. Here we consider the case of the vector field φ^μ in the presence of the additional scalar field λ and derive the free propagator of the system. The Lagrangian of the (φ_μ, λ) system can be written as

$$\mathcal{L}^{\mathrm{v}} = -\frac{1}{2} \left(\partial^\mu \varphi_\nu \right) \left(\partial_\mu \varphi^\nu \right) + \frac{1}{2} m^2 \varphi_\nu \varphi^\nu + a\, m \left(\partial_\nu \varphi^\nu \right) \lambda - \frac{1}{2} a^2\, m^2\, \lambda^2, \tag{7}$$

where φ_μ is a vector and λ is an auxiliary scalar field and a is an arbitrary free parameter. Independent variations of the vector and auxiliary fields produce two equations of motion

$$(\Box + m^2)\varphi_\mu - a\, m \partial_\mu \lambda = 0,$$
$$a\, m\, \partial_\mu \varphi^\mu - a^2 m^2\, \lambda = 0. \tag{8}$$

Diagonalization of the system (8) leads to the Proca equation for the vector field φ_μ whereas the auxiliary field λ vanishes. Although $\lambda = 0$ the propagator of the system always contains a component associated with the auxiliary field and $\varphi_\mu - \lambda$ mixing terms.

To obtain a propagator for the system of fields (φ_μ, λ) it is convenient to rewrite eq. (8) in the matrix form

$$\Lambda^{\mathrm{v}}_{\{\mu\nu\}} \Phi^{\mathrm{v}\{\nu\}} = 0, \tag{9}$$

where

$$\Lambda^{\mathrm{v}}_{\{\mu\nu\}} = \begin{pmatrix} (\Box + m^2)\mathrm{g}_{\mu\nu} & -a\, m \partial_\mu \\ a\, m\, \partial_\nu & -a^2 m^2 \end{pmatrix} \text{ and } \Phi^{\mathrm{v}\{\nu\}} = \begin{pmatrix} \varphi^\nu \\ \lambda \end{pmatrix}. \tag{10}$$

Since the system contains vector and scalar degrees of freedom the Lorentz indices in curly brackets of eqs. (9,10) are associated with corresponding tensor and vector elements of $\Lambda^{\mathrm{v}}_{\{\mu\nu\}}$ and $\Phi^{\mathrm{v}}_{\{\nu\}}$.

The inverse operator (propagator) can be obtained as a solution of the following equation

$$\Lambda^{\mathrm{v}}_{\{\rho}{}^{\mu\}} \, G^{\mathrm{v}}_{\{\mu\nu\}} = I^{\mathrm{v}}_{\{\mu\nu\}} \, \delta^4(x - x'), \tag{11}$$

where the propagator $G^{\mathrm{v}}_{\{\mu\nu\}}$ and the unit matrix $I^{\mathrm{v}}_{\{\mu\nu\}}$ are defined as

$$G^{\mathrm{v}}_{\{\mu\nu\}} = \begin{pmatrix} G^{(\varphi\varphi)}_{\mu\nu} & G^{(\varphi\lambda)}_\mu \\ G^{(\lambda\varphi)}_\nu & G^{(\lambda\lambda)} \end{pmatrix} \quad \text{and} \quad I^{\mathrm{v}}_{\{\mu\nu\}} = \begin{pmatrix} \mathrm{g}_{\mu\nu} \\ 1 \end{pmatrix}.$$

The four components of the matrix $G^{\mathrm{v}}_{\{\mu\nu\}}$ have simple physical meanings: $G^{(\varphi\varphi)}_{\mu\nu}$ and $G^{(\lambda\lambda)}$ stand for the propagator of the purely vector and auxiliary scalar fields, correspondingly, whereas the nondiagonal $G^{(\varphi\lambda)}_\mu$ and $G^{(\lambda\varphi)}_\nu$ terms are associated with the $\varphi - \lambda$ mixing. The solution of eq. (11) in the momentum space is

$$G^{\mathrm{v}}_{\{\mu\nu\}}(p) = \begin{pmatrix} \dfrac{\left(-\mathrm{g}_{\mu\nu} + \dfrac{p_\mu p_\nu}{m^2}\right)}{p^2 - m^2} & \mathrm{i}\dfrac{p_\mu}{a\,m^3} \\[4ex] -\mathrm{i}\dfrac{p_\nu}{a\,m^3} & \dfrac{(p^2 - m^2)}{a^2\,m^4} \end{pmatrix}. \tag{12}$$

The pole $(p^2 - m^2)^{-1}$ appears only at the vector component $G^{(\varphi\varphi)}_{\mu\nu}(p)$; this term completely coincides with the corresponding expression well known from quantum field theory. The remaining terms in the propagator depend on the free parameter a associated with the auxiliary field λ. From eq. (12) one can conclude that $G^{(\lambda\lambda)}$ gives contributions only off-shell $p^2 \neq m^2$. Note that the scalar component of the propagating vector field φ_μ mixes with the λ field which leads to the appearance of the finite non-diagonal components in the propagator eq. (12).

Despite the complications related to the introduction of the auxiliary field the description in terms of (φ_μ, λ) system is completely equivalent to the conventional description in terms of the pure vector field. It implies that physical observables do not depend on the free parameter a appearing in the the full propagator (12). For the free fields this conclusion immediately follows from the fact that the auxiliary field can be excluded from the upper equation (8). It also holds true in the case of interacting fields provided there is no coupling to auxiliary degrees of freedom.

IV. PROPAGATOR FOR THE FREE SPIN-$\frac{5}{2}$ FIELD.

Similar to the procedure described in Section III it is convenient to rewrite the set of equations $(3, 4)$ in matrix form

$$\begin{pmatrix} \Lambda^{(\psi\psi)}_{\mu\nu;\rho\sigma}(p) & m\,c\,\mathrm{g}_{\mu\nu} \\ c\,m\,\mathrm{g}_{\rho\sigma} & \Lambda^{(\xi\xi)}(p) \end{pmatrix} \begin{pmatrix} \psi^{\rho\sigma}(p) \\ \xi(p) \end{pmatrix} = 0, \tag{13}$$

where $\Lambda^{(\psi\psi)}_{\mu\nu;\rho\sigma}(p) = \Lambda^{(1)}_{\mu\nu;\rho\sigma}(p) + \Lambda^{(2)}_{\mu\nu;\rho\sigma}(p)$ and $\Lambda^{(\xi\xi)}(p) = B(a, b, c)\,(\not{p} + 3m)$. While the auxiliary field vanishes on-shell the full propagator should also contain an off-shell part related with the auxiliary field $\xi(x)$. Hence, the full propagator of the system is

$$\mathcal{G}^{\{\tau\lambda;\rho\sigma\}}(p) = \begin{pmatrix} G^{\tau\lambda;\rho\sigma}_{(\psi\psi)}(p) & G^{\tau\lambda;}_{(\psi\xi)}(p) \\ G^{;\rho\sigma}_{(\xi\psi)}(p) & G_{(\xi\xi)}(p) \end{pmatrix} \tag{14}$$

and satisfies the equation

$$\begin{pmatrix} \Lambda^{(\psi\psi)}_{\mu\nu;\tau\lambda}(p) & m\,c\,\mathrm{g}_{\mu\nu} \\ c\,m\,\mathrm{g}_{\tau\lambda} & \Lambda^{(\xi\xi)}(p) \end{pmatrix} \begin{pmatrix} G^{\tau\lambda;\rho\sigma}_{(\psi\psi)}(p) & G^{\tau\lambda;}_{(\psi\xi)}(p) \\ G^{;\rho\sigma}_{(\xi\psi)}(p) & G_{(\xi\xi)}(p) \end{pmatrix} = \begin{pmatrix} I^{\rho\sigma}_{\mu\nu} & 0 \\ 0 & 1 \end{pmatrix}, \tag{15}$$

where $I^{\rho\sigma}_{\mu\nu} = \mathrm{g}^\rho_\mu \mathrm{g}^\sigma_\nu + \mathrm{g}^\sigma_\mu \mathrm{g}^\rho_\nu$. The diagonal terms $G^{\mu\nu;\rho\sigma}_{(\psi\psi)}(p)$ and $G_{(\xi\xi)}(p)$ are related to fields ψ and ξ respectively whereas the non-diagonal ones stand for mixing between the auxiliary spinor field and the 'off-shell' spin-$\frac{1}{2}$ component of the spin-$\frac{5}{2}$ field.

The propagator of the spin-$\frac{5}{2}$ field $G^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(p) = G^{(\psi\psi)}_{\mu\nu;\rho\sigma}(p)$ is obtained as a solution of the set of equations

$$\Lambda^{(\psi\psi)}_{\mu\nu;\tau\lambda}(p)\,G^{\tau\lambda;\rho\sigma}_{(\psi\psi)}(p) + m\,c\,\mathrm{g}_{\mu\nu}\,G^{;\rho\sigma}_{(\xi\psi)}(p) \;=\; I^{\rho\sigma}_{\mu\nu},$$
$$c\,m\,\mathrm{g}_{\tau\lambda}G^{\tau\lambda;}_{(\psi\xi)}(p) + \Lambda^{(\xi\xi)}(p)\,G_{(\xi\xi)}(p) \;=\; 1. \tag{16}$$

In the literature one sometimes encounters a propagator defined as

$$G'_{\rho\sigma;\tau\delta}(p) = \frac{\slashed{p}+m}{p^2-m^2+i\epsilon}\mathcal{P}^{\frac{5}{2}}_{\rho\sigma;\tau\delta}(p), \tag{17}$$

where $\mathcal{P}^{\frac{5}{2}}_{\rho\sigma;\tau\delta}(p)$ is a spin-$\frac{5}{2}$ projection operator in the spinor-tensor representation (A1). However, the quatity defined above does not have an inverse and therefore cannot obey eqs. (16) for any choice of the free parameters. This can be shown by replacing the $G^{\tau\lambda;\rho\sigma}_{(\psi\psi)}(p)$ in the upper equation (16) by expression from eq. (17) and multiplying the both sides of the resulting equation from the right by a projection operator $\mathcal{P}^{\frac{3}{2}}_{22;\rho\sigma,\tau\delta}(p)$. Using the general properties of projection operators eq. (A2) the obtained expression reduces to

$$m\,c\,\mathrm{g}_{\mu\nu}\,G^{\rho\sigma}_{(\xi\psi)}(p)\mathcal{P}^{\frac{3}{2}}_{22;\rho\sigma,\tau\delta}(p) \;=\; 2\mathcal{P}^{\frac{3}{2}}_{22;\mu\nu,\tau\delta}(p). \tag{18}$$

This leads to a contradiction: from eq. (18) follows that $G^{\rho\sigma}_{(\xi\psi)}(p)\mathcal{P}^{\frac{3}{2}}_{22;\rho\sigma,\tau\delta}(p)$ cannot be zero but multiplying the both sides of the same equation by $\mathrm{g}^{\mu\nu}$ and using the property $\mathrm{g}^{\mu\nu}\mathcal{P}^{\frac{3}{2}}_{22;\mu\nu,\tau\delta}(p) = 0$ one can draw an opposite conclusion. Hence, the quantity defined in eq. (17) does not obey eqs. (16) and cannot be a spin-$\frac{5}{2}$ propagator.

The solution of equation (16) where the parameters a and b are kept to be free is tedious. Here we confine ourselves by looking for a solution with a specific choice of the parameters $a = -1$, $b = -1$ whereas c is kept to be arbitrary. The independence of $G^{\tau\lambda;\rho\sigma}_{(\psi\psi)}(p)$ on the latter parameter signifies that the auxiliary field does not contribute to the physical observables. We discuss this issue in Section V. With this specific choice of the free parameters the resulting equations are

$$[(\slashed{p}-m)(\mathrm{g}_{\mu\tau}\mathrm{g}_{\nu\lambda}+\mathrm{g}_{\mu\lambda}\mathrm{g}_{\nu\tau}) - (\gamma_\mu p_\nu \mathrm{g}_{\lambda\tau}+\gamma_\nu p_\mu \mathrm{g}_{\lambda\tau}+\gamma_\lambda p_\tau \mathrm{g}_{\mu\nu}+\gamma_\tau p_\lambda \mathrm{g}_{\mu\nu}) + (\slashed{p}+m)\,\mathrm{g}_{\mu\nu}\mathrm{g}_{\tau\lambda}$$
$$-(\gamma_\mu p_\lambda \mathrm{g}_{\nu\tau}+\gamma_\nu p_\lambda \mathrm{g}_{\mu\tau}+\gamma_\mu p_\tau \mathrm{g}_{\nu\lambda}+\gamma_\nu p_\tau \mathrm{g}_{\mu\lambda}+\gamma_\lambda p_\mu \mathrm{g}_{\nu\tau}+\gamma_\tau p_\mu \mathrm{g}_{\nu\lambda}+\gamma_\lambda p_\nu \mathrm{g}_{\mu\tau}+\gamma_\tau p_\nu \mathrm{g}_{\mu\lambda})$$
$$+(\gamma_\mu \slashed{p}\gamma_\lambda \mathrm{g}_{\nu\tau}+\gamma_\nu \slashed{p}\gamma_\lambda \mathrm{g}_{\mu\tau}+\gamma_\mu \slashed{p}\gamma_\tau \mathrm{g}_{\nu\lambda}+\gamma_\nu \slashed{p}\gamma_\tau \mathrm{g}_{\mu\lambda}) + m\,(\gamma_\mu \gamma_\lambda \mathrm{g}_{\nu\tau}+\gamma_\nu \gamma_\lambda \mathrm{g}_{\mu\tau}$$
$$+\gamma_\mu \gamma_\tau \mathrm{g}_{\nu\lambda}+\gamma_\nu \gamma_\tau \mathrm{g}_{\mu\lambda})]G^{\tau\lambda;\rho\sigma}_{(\psi\psi)}(p) + m\,c\,\mathrm{g}_{\mu\nu}\,G^{\rho\sigma}_{(\xi\psi)}(p) = \mathrm{g}^\rho_\mu\,\mathrm{g}^\sigma_\nu + \mathrm{g}^\sigma_\mu\,\mathrm{g}^\rho_\nu,$$
$$m\,\mathrm{g}_{\tau\lambda}G^{\tau\lambda}_{(\psi\xi)}(p) - \frac{6\,c^2}{5}\,(\slashed{p}+3m)\,(p)\,G_{(\xi\xi)}(p) = 1. \tag{19}$$

The obtained spin-$\frac{5}{2}$ propagator $G^{\mu\nu;\rho\sigma}_{\frac{5}{2}}(p) = G^{\mu\nu;\rho\sigma}_{(\psi\psi)}(p)$ can be written as a decomposition in terms of projection operators as follows [7]

$$G^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(p) \;=\; \frac{1}{p^2-m^2}\left((\slashed{p}+m)\,\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(p) - \frac{p^2-m^2}{m^2}\left(\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p)+\mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p)\right)\right), \tag{20}$$

where $\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p)$ and $\mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p)$ stand for the contributions from the spin-$\frac{3}{2}$ and -$\frac{1}{2}$ sector of the spinor-vector representation, (see Appendix C). As expected the propagator for the spin-$\frac{5}{2}$ itself does not depend on the parameter c related to the spinor field ξ. This observation also holds for arbitrary values of a and b in eq. (16) which we have checked by explicit calculations. The obtained propagator has a pole associated with the spin-$\frac{5}{2}$ part and so called 'off-shell' non-pole contributions coming from the lower spin components $\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p)$ and $\mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p)$.

V. COUPLING TO HIGHER SPIN FERMIONS

In the case of the spin-$\frac{5}{2}$ field in the spinor-tensor representation we deal with a system $(\psi_{\mu\nu},\xi)$ which contains auxiliary degrees of freedom. One can raise a question whether the unphysical degrees of freedom could be eliminated

from physical observables. Here we consider a simple case of spin-$\frac{5}{2}$ resonance contribution to πN scattering which is valid for applications in hadron physics. The corresponding $\pi N N^*_{\frac{5}{2}}$ coupling can be chosen as follows

$$\mathcal{L}_I = \frac{g_{\pi N N^*}}{4m_\pi^2} \left(\bar{\psi}_N(x), 0\right) \Gamma_{\mu\nu;\rho\sigma} \left[\hat{P}_{(\psi\psi)} \begin{pmatrix} \psi^{\rho\sigma} \\ \xi \end{pmatrix}\right] \partial^\mu \partial^\nu \pi(x) + \text{h.c.},$$

where the nucleon field is written as $\left(\bar{\psi}_N(x), 0\right)$ which implies the absence of auxiliary fields in the final state. The operator

$$\hat{P}_{(\psi\psi)} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

projects out the spin-$\frac{5}{2}$ field and ensures that there is no coupling to ξ. Hence, only the spin-$\frac{5}{2}$ component of the propagator $G^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(p)$, eq. (20), contributes to physical observables at any order of perturbation theory. In Section III we have demonstrated that the inclusion of auxiliary degrees of freedom in the vector field does not affect the physical observables. To our knowledge this statement is not generally proven for the $(\psi_{\mu\nu}, \xi)$ system beyond the pertubation expansion. The reason is that the equation of the motion for massive spin-$\frac{5}{2}$ field in the spinor-tensor representation is defined only in the presence of an auxiliary field. This is unlike the case of the vector field where auxiliary degrees of freedom can be removed by proper field transformations. Note that these degrees of freedom contribute due to $\psi_{\mu\nu} - \xi$ mixing. This mixing takes place only between the spin-$\frac{1}{2}$ sector of the spinor-tensor and the auxiliary spinor fields, as pointed out in the previous Section. One may therefore hope that the use of a coupling which suppresses the spin-$\frac{1}{2}$ contributions would also prevent the appearance of the auxiliary degrees of freedom in the physical observables in the non perturbative regime.

A possibility to remove unwanted degrees of freedom in a special case of the spin-$\frac{3}{2}$ fields has been demonstrated in [3, 13, 14]. The idea is based on the observation that the Lagrangian of the Rarita-Schwinger fields for a specific choice of the parameter $A = -1$ maintains gauge-invariance in the massless limit. Therefore the use of a gauge-invariant coupling suppresses the contribution from the lower spin sector.

Guided by the results obtained in the spin-$\frac{3}{2}$ Rarita-Schwinger theory one could expect that the lower spin terms of the spin-$\frac{5}{2}$ propagator eq. (20) do not contribute to the physical observables as long as a gauge invariant coupling is used. This however is not generally true for the spin-$\frac{5}{2}$ fields in the spinor-tensor representation. Such a conclusion can be drawn from the fact that the Lagrangian eq. (2) is not invariant under the gauge transformations $\psi_{\mu\nu} \to \psi_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$ in a massless limit at any choice of parameters a and b. We show this more explicitly by exploring a general structure of the gauge-invariant vertex function on the example of the πN scattering amplitude in the leading order of the perturbation expansion. The amplitude can in general be written in the form

$$\mathcal{M} \sim \bar{u}_N(p') \left[\Gamma^{\mu\nu;\rho\sigma}(q) \, G^{\frac{5}{2}}_{\rho\sigma;\alpha\beta}(q) \, \Gamma^{\dagger\,\alpha\beta;\lambda\tau}(q) \right] u_N(p) \, k'_\mu \, k'_\nu \, k_\lambda \, k_\tau, \tag{21}$$

where p (k) and $p'(k')$ are momenta of the initial and final nucleon(pion) correspondingly; q stands for the momentum of the resonance and depends on the channel (s- or u-) of interest. The gauge-invariant coupling to the spin-$\frac{5}{2}$ field imposes the following constraint on the vertex function $q_\rho \, \Gamma^{\mu\nu;\rho\sigma}(q) = q_\sigma \, \Gamma^{\mu\nu;\rho\sigma}(q) = 0$. For the transition matrix to be free from any contribution from the lower spin sector the expression in the square brackets in eq. (21) should be proportional to the spin-$\frac{5}{2}$ projection operator

$$\left[\Gamma^{\mu\nu;\rho\sigma}(q) \, G^{\frac{5}{2}}_{\rho\sigma;\alpha\beta}(q) \, \Gamma^{\dagger\,\alpha\beta;\lambda\tau}(q) \right] \sim \mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q). \tag{22}$$

This gives an additional constraint $q_\mu \, \Gamma^{\mu\nu;\rho\sigma}(q) = q_\nu \, \Gamma^{\mu\nu;\rho\sigma}(q) = 0$. The vertex function can be decomposed in terms of the spin projection operators. There are only three operators $\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q)$, $\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$, and $\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ which fulfill the properties $q^\mu \, \mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q) = 0$, $q^\nu \, \mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q) = 0$, etc. Hence, the decomposition can be written as follows

$$\Gamma_{\mu\nu;\rho\sigma}(q) = \alpha_1(\not{q}) \, \mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q) + \alpha_2(\not{q}) \, \mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q) + \alpha_3(\not{q}) \, \mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q), \tag{23}$$

where the coefficients of the decomposition $\alpha_1(\not{q})$, $\alpha_2(\not{q})$, and $\alpha_3(\not{q})$ are polynomials of m and $\not{q}$. Note, that $\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q)$, $\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$, and $\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ commute with $\not{q}$. The spin-$\frac{5}{2}$ propagator can also be decomposed in terms of the spin projection operators. Due to the orthogonality properties of the projection operators only those terms in $G^{\frac{5}{2}}_{\rho\sigma;\alpha\beta}(q)$ contribute to the matrix element eq. (21) which contain $\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q)$, $\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$, and $\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ operators. If the

parameters a and b in eq. (15, 16) could be chosen in such a way that the propagator does not contain the $\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ and $\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ operators the lower spin contributions to the matrix element would be suppressed. Such a situation is realized in the spin-$\frac{3}{2}$ Rarita-Schwinger theory for the special choice of the free parameter $A = -1$, see [4] for discussion. For the spin-$\frac{5}{2}$ fields eq. (4) the contribution of the $\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ projector can be suppressed by choosing $a = -1$. As we already mentioned in Section II this parameter is associated with both spin-$\frac{3}{2}$ and -$\frac{1}{2}$ degrees of freedom whereas b regulates only the spin-$\frac{1}{2}$ ones. Indeed the expression eq. (C1) derived for $a = -1$, $b = -1$ does not have the $\mathcal{P}^{\frac{3}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ projector. One can ask whether $\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$ can also be removed from the free propagator. The general conclusion is that the term $G^{\tau\lambda;\rho\sigma}_{(\psi\psi)}(p)$ being a solution of eq.(16) always has contributions from $\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(q)$. We have checked this by explicit calculation for arbitrary values of parameter b. This conclusion is ultimately linked to the fact that the Lagrangian of the free spin-$\frac{5}{2}$ fields (2) does not maintain gauge-invariance in the massless limit. The same conclusion has been also drawn in [7]. Therefore one can never remove the corresponding degrees of freedom from the transition matrix eq. (21) provided the vertex function is written in the form of eq. (23).

The solution to the problem has been suggested in [4] where authors suggested to utilize the Rarita-Schwinger condition $\gamma_\mu \psi^{\mu\nu} = \gamma_\nu \psi^{\mu\nu} = 0$ to constrain the interaction vertex. As a result the interaction vertex fulfills the condition $\gamma \cdot \Gamma = \Gamma \cdot \gamma = 0$. Applying this constraint to the decomposition eq. (23) one can see that only the $\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q)$ projector obeys the desired property and the decomposition reduces to

$$\Gamma_{\mu\nu;\rho\sigma}(q) = \alpha_1(\slashed{q}) \, \mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q). \tag{24}$$

Since the vertex function should be free from any singularities the minimal power of $\slashed{q}$ in the function $\alpha_1(\slashed{q})$ should be of the fourth order. Then, the most simple coupling can be written as follows

$$\mathcal{L}^{\frac{5}{2}}_{\pi NN^*} = \frac{\mathrm{g}_{\pi NN^*}}{m_\pi^2 m_R^4} \, \bar{\psi}_N(x) \left[\Box^2 \mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(\partial) \psi_{N^*}^{\mu\nu} \right] \partial^\mu \partial^\nu \pi(x) + \text{h.c.} \tag{25}$$

The use of $\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(\partial)$ ensures that only the spin-$\frac{5}{2}$ part of the propagator contributes and the d'Alembert-operator squared guarantees that no other singularities except the mass pole term $(p^2 - m^2)^{-1}$ appear in the amplitude. As a results the physical observables no longer depend on the arbitrary parameters a and b of the free Lagrangian. The πN scattering amplitude eq. (21) then reads

$$\mathcal{M} = \left(\frac{\mathrm{g}_{\pi NN^*}}{m_\pi^2} \right)^2 \bar{u}_N(p') \left[\left(\frac{q^2}{m_R^2} \right)^4 \mathcal{P}^{\frac{5}{2}}_{\mu\nu;\lambda\tau}(q) \right] u_N(p) \, k'^\mu \, k'^\nu \, k^\lambda \, k^\tau, \tag{26}$$

The coupling in eqs. (25) can be generalized for the fields $\psi_{N^*}^{\{\rho...\sigma\}}$ of the arbitrary spin J as

$$\mathcal{L}^J_{\pi NN^*} = \frac{\mathrm{g}_{\pi NN^*}}{(m_\pi)^{\frac{1}{2}(J-1/2)}} \, \bar{\psi}_N(x) \left[\left(\frac{\Box}{m_R^2} \right)^{(J-\frac{1}{2})} \mathcal{P}^J_{\{\mu...\delta;\rho...\sigma\}}(\partial) \psi_{N^*}^{\{\rho...\sigma\}} \right] \{\partial^\mu\}...\{\partial^\delta\} \pi(x) + \text{h.c.}, \tag{27}$$

where the number of indices assigned to $\psi_{N^*}^{\{\rho...\sigma\}}$ and $\mathcal{P}^J_{\{\mu...\nu;\rho...\sigma\}}(\partial)$ depends on the chosen representation. The coupling constructed in eqs. (26, 27) ensures that physical observables do not depend on the free parameters of the theory.

VI. SUMMARY

In summary, we have investigated the general properties of the free spin-$\frac{5}{2}$ fields in the spinor-tensor representation. The Lagrangian is written in terms of spin-$\frac{5}{2}$ and auxiliary fields $(\psi_{\mu\nu}, \xi)$ and coincides with that suggested in the literature for a specific choice of free parameters. We demonstrate that the Lagrangian in general depends on three arbitrary parameters; two of them are associated with the lower spin-$\frac{3}{2}$ and -$\frac{1}{2}$ sector of the theory whereas the third one is linked to the auxiliary field ξ.

We deduce a free propagator of the system which is given by a 2×2 matrix in the $(\psi_{\mu\nu}, \xi)$ space. The diagonal elements stand for the propagation of the spin-$\frac{5}{2}$ and ξ fields whereas the non-diagonal ones correspond to $\psi_{\mu\nu} - \xi$ mixing. The mixing takes place between the spin-$\frac{1}{2}$ sector of the spinor-tensor representation and an auxiliary spinor

field. While the free propagator includes auxiliary degrees of freedom they do not contribute to the physical observables calculated within the perturbation theory provided there is no coupling to ξ.

As an application to hadron physics calculations, the interaction involving $(\psi_{\mu\nu}, \xi)$ is discussed for the example of the $\pi N N^*_{\frac{5}{2}}$ coupling. The pure spin-$\frac{5}{2}$ propagator contains non-pole terms which contribute in the whole kinematical region. As we demonstrate invariance under gauge transformations is not enough to remove these contributions. This is ultimately related to the fact that the free Lagrangian of the $(\psi_{\mu\nu}, \xi)$ system does not maintain gauge invariance in the massless limit for any choice of the free parameters. The desired result can, however, be obtained by constructing a coupling with higher order derivatives. In the latter case the amplitude of the πN scattering does not depend on the arbitrary parameters of the free Lagrangian. The suggested coupling is generalized to the Rarita-Schwinger fields of any half-integer spin.

Acknowledgments

APPENDIX A: SPIN PROJECTION OPERATORS FOR THE SPINOR-TENSOR REPRESENTATION

The spin projection operators are taken from [7]. In the momentum space they are given by

$$
\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q) = \frac{1}{2}\left(\mathcal{P}^1_{\mu\rho}\,\mathcal{P}^1_{\nu\sigma} + \mathcal{P}^1_{\mu\sigma}\,\mathcal{P}^1_{\nu\rho}\right) - \frac{1}{5}\,\mathcal{P}^1_{\mu\nu}\,\mathcal{P}^1_{\rho\sigma}
$$
$$
- \frac{1}{10}\left(\mathcal{P}^1_{\mu}\,\mathcal{P}^1_{\rho}\,\mathcal{P}^1_{\nu\sigma} + \mathcal{P}^1_{\nu}\,\mathcal{P}^1_{\rho}\,\mathcal{P}^1_{\mu\sigma} + \mathcal{P}^1_{\mu}\,\mathcal{P}^1_{\sigma}\,\mathcal{P}^1_{\nu\rho} + \mathcal{P}^1_{\nu}\,\mathcal{P}^1_{\sigma}\,\mathcal{P}^1_{\mu\rho}\right),
$$

$$
\mathcal{P}^{\frac{3}{2}}_{11;\mu\nu;\rho\sigma}(q) = \frac{1}{2}\left(\mathcal{P}^1_{\mu\rho}\,\mathcal{P}^0_{\nu\sigma} + \mathcal{P}^1_{\nu\rho}\,\mathcal{P}^0_{\mu\sigma} + \mathcal{P}^1_{\mu\sigma}\,\mathcal{P}^0_{\nu\rho} + \mathcal{P}^1_{\nu\sigma}\,\mathcal{P}^0_{\mu\rho}\right) - \frac{1}{6\,q^2}\,\mathcal{O}_{\mu\nu}\,\mathcal{O}_{\rho\sigma},
$$

$$
\mathcal{P}^{\frac{3}{2}}_{22;\mu\nu;\rho\sigma}(q) = \frac{1}{10}\left(\mathcal{P}^1_{\mu}\,\mathcal{P}^1_{\rho}\,\mathcal{P}^1_{\nu\sigma} + \mathcal{P}^1_{\nu}\,\mathcal{P}^1_{\rho}\,\mathcal{P}^1_{\mu\sigma} + \mathcal{P}^1_{\mu}\,\mathcal{P}^1_{\sigma}\,\mathcal{P}^1_{\nu\rho} + \mathcal{P}^1_{\nu}\,\mathcal{P}^1_{\sigma}\,\mathcal{P}^1_{\mu\rho}\right) - \frac{2}{15}\,\mathcal{P}^1_{\mu\nu}\,\mathcal{P}^1_{\rho\sigma},
$$

$$
\mathcal{P}^{\frac{3}{2}}_{21;\mu\nu;\rho\sigma}(q) = -\mathcal{P}^{\frac{3}{2}}_{12;\rho\sigma;\mu\nu}(q) = \frac{1}{2\sqrt{5}\,q^2}\left(q_\rho \mathcal{P}^1_{\mu}\,\mathcal{P}^1_{\nu\sigma} + q_\rho \mathcal{P}^1_{\nu}\,\mathcal{P}^1_{\mu\sigma} + q_\sigma \mathcal{P}^1_{\mu}\,\mathcal{P}^1_{\nu\rho} + q_\sigma \mathcal{P}^1_{\nu}\,\mathcal{P}^1_{\mu\rho}\right)\slashed{q}
$$
$$
- \frac{1}{3\sqrt{5}\,q^2}\,\mathcal{P}^1_{\mu\nu}\,\mathcal{O}_{\rho\sigma}\,\slashed{q},
$$

$$
\mathcal{P}^{\frac{1}{2}}_{11;\mu\nu;\rho\sigma}(q) = \mathcal{P}^0_{\mu\nu}\,\mathcal{P}^0_{\rho\sigma},
$$

$$
\mathcal{P}^{\frac{1}{2}}_{22;\mu\nu;\rho\sigma}(q) = \frac{1}{3}\,\mathcal{P}^1_{\mu\nu}\,\mathcal{P}^1_{\rho\sigma},
$$

$$
\mathcal{P}^{\frac{1}{2}}_{33;\mu\nu;\rho\sigma}(q) = \frac{1}{6\,q^2}\,\mathcal{O}_{\mu\nu}\,\mathcal{O}_{\rho\sigma},
$$

$$
\mathcal{P}^{\frac{1}{2}}_{21;\mu\nu;\rho\sigma}(q) = \mathcal{P}^{\frac{1}{2}}_{12;\rho\sigma;\mu\nu}(q) = \frac{1}{\sqrt{3}}\,\mathcal{P}^1_{\mu\nu}\,\mathcal{P}^0_{\rho\sigma},
$$

$$
\mathcal{P}^{\frac{1}{2}}_{31;\mu\nu;\rho\sigma}(q) = -\mathcal{P}^{\frac{1}{2}}_{13;\rho\sigma;\mu\nu}(q) = \frac{1}{\sqrt{6}\,q^2}\,\mathcal{O}_{\mu\nu}\,\mathcal{P}^0_{\rho\sigma}\,\slashed{q},
$$

$$
\mathcal{P}^{\frac{1}{2}}_{23;\mu\nu;\rho\sigma}(q) = -\mathcal{P}^{\frac{1}{2}}_{32;\rho\sigma;\mu\nu}(q) = \frac{-1}{3\sqrt{2}\,q^2}\,\mathcal{O}_{\rho\sigma}\,\mathcal{P}^1_{\mu\nu}\,\slashed{q}, \tag{A1}
$$

where operators $\mathcal{P}^1_{\mu\nu}$, $\mathcal{P}^0_{\mu\nu}$, $\mathcal{P}^1_{\mu}$, and $\mathcal{O}_{\mu\nu}$ are defined as

$$\mathcal{P}^1_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \qquad \mathcal{P}^1_\mu = \mathcal{P}^1_{\mu\nu}\gamma^\nu,$$

$$\mathcal{P}^0_{\mu\nu} = \frac{q_\mu q_\nu}{q^2}, \qquad \mathcal{O}_{\mu\nu} = \mathcal{P}^1_\mu q_\nu + q_\mu \mathcal{P}^1_\nu.$$

The projection operators eq. (A1) fulfill the following properties: they satisfy orthogonality conditions

$$\mathcal{P}^{J}_{ii;\mu\nu}{}^{;\tau\lambda}(q)\,\mathcal{P}^{J'}_{ii;\tau\lambda;\rho\sigma}(q) = \delta_{JJ'}\mathcal{P}^{J}_{ii;\mu\nu;\rho\sigma}(q) \tag{A2}$$

and the sum rules

$$\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(q) + \sum_{i=1}^{2}\mathcal{P}^{\frac{3}{2}}_{ii;\mu\nu;\rho\sigma}(q) + \sum_{i=1}^{3}\mathcal{P}^{\frac{1}{2}}_{ii;\mu\nu;\rho\sigma}(q) = \frac{1}{2}(g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}). \tag{A3}$$

APPENDIX B: LAGRANGIAN FOR THE FREE SPIN-$\frac{5}{2}$ FIELD

The functions $F_1(a)$, $F_2(a)$, $G_1(a,b)$, $G_2(a,b)$, and $B(a,b,c)$ of the free real parameters a, b, and c used in the definition of the Lagrangian eq. (2) read

$$
\begin{aligned}
F_1(a) &= \frac{1}{4}(5a^2 + 2a + 1), \\
F_2(a) &= \frac{1}{8}(15a^2 + 10a + 3), \\
G_1(a,b) &= \frac{5a^4 - 12a^3 - 20a^2 - 8a - 4b^2 - 4\,b\,(7a^2 + 6\,a + 1) - 1}{2(3a+1)^2}, \\
G_2(a,b) &= \frac{-15a^4 + 18a^2 + 8a + 12b^2 + 6\,b\,(5a^2 + 6\,a + 1) + 1}{2(3a+1)^2}, \\
B(a,b,c) &= -\frac{24\,c^2\,(3a+1)^2}{5(5\,a^2 + 6\,a + 4\,b + 1)^2}.
\end{aligned}
\tag{B1}
$$

Using the variational principle one obtains two equations of motion eqs. (3, 4). Here we show that all Rarita-Schwinger constraints [1] can be obtained from these equations. By multiplying the eq. (3) by $g^{\mu\nu}$, $\gamma^\mu p^\nu$ and $p^\mu p^\nu$ and making summation we get

$$
\begin{aligned}
\Big(g_{\rho\sigma}(\not{p}(b - 2F_1(a) + 2G_1(a,b) + 1) + m(\,2F_2(a) + 2G_2(a,b) - 1)) \\
+ 2(\gamma_\sigma p_\rho + \gamma_\rho p_\sigma)(a + b + F_1(a)))\psi^{\rho\sigma} + 2\,c\,m\xi = 0,
\end{aligned}
\tag{B2}
$$

$$
\begin{aligned}
\Big((2a + 5\,b + 2F_1(a) + G_1(a,b))\,p^2 g_{\rho\sigma} + m\not{p}\,(G_2(a,b) - 2F_2(a))g_{\rho\sigma} \\
+ (-m + 6\,mF_2(a) + (b - 1 + 4F_1(a)\,)\not{p})(\gamma_\rho p_\sigma + \gamma_\sigma p_\rho) \\
+ (12a + 4)\,p^\rho p^\sigma)\psi^{\rho\sigma} + c\,m\not{p}\,\xi = 0,
\end{aligned}
\tag{B3}
$$

$$
\begin{aligned}
\Big(2((2a+1)\not{p} - m)p^\rho p^\sigma + (p^2(2a + b + 2F_1(a)) + 2mF_2(a)\not{p})(\gamma^\rho p^\sigma + \gamma^\sigma p^\rho) \\
+ p^2((2b + G_1(a,b))\not{p} + mG_2(a,b))g^{\rho\sigma})\psi_{\rho\sigma} + c\,m\,p^2\,\xi = 0,
\end{aligned}
\tag{B4}
$$

correspondingly. Expessing $(\gamma_\sigma p_\rho + \gamma_\rho p_\sigma)\psi^{\rho\sigma}$ from the first equation (B2) and substituing it in to eqs. (B3, B4) we obtain

$$
\begin{aligned}
\Big(4\,[3a+1][a + b + F_1(a)]\,p^\rho p^\sigma + \frac{1}{2}\big(-[6F_2(a) - 1][2F_2(a) + 2G_2(a,b) - 1]m^2 + 2\not{p}\,[b + G_1(a,b) - 2F_2(a)\\
(a + 3b + 3G_1(a,b) + 1) + F_1(a)(1 - 3G_2(a,b)) + (a+1)G_2(a,b)]m + p^2[4\,a^2 + 14\,a\,b + 9\,b^2 + 12F_1^2(a)\\
+ 2F_1(a)(4a + 6b - 3G_1(a,b) - 3) + 2\,(a+1)\,G_1(a,b) + 1]\big)g^{\rho\sigma}\Big)\psi_{\rho\sigma}\\
+ c\,m\,\big([1 - 6F_2(a)]m + \not{p}\,[a - 3F_1(a) + 1]\big)\xi = 0 \quad \text{(B5)}
\end{aligned}
$$

$$\Big(2[a + b + F_1(a)][(2a + 1)\not{p} - m]p^\rho p^\sigma + \frac{1}{2}\big(- 2F_2(a)[2F_2(a) + 2G_2(a, b) - 1]m^2\not{p}$$

$$-p^2(\not{p}(-3b^2 - 2ab + b - 4F_1^2(a) + 2a + 2a\,G_1(a, b) + F_1(a)(-4a - 4b + 2G_1(a, b) + 2))$$

$$+m(2G_2(a, b)a - 2a - b + F_2(a)(4a + 4b + 4G_1(a, b) + 2) + 2F_1(a)(G_2(a, b) - 1))))\mathrm{g}^{\rho\sigma}\Big)\psi_{\rho\sigma}$$

$$+c\,m\,\big(- [a + F_1(a)]\,p^2 - 2mF_2(a)\,\not{p}\,\big)\xi = 0. \tag{B6}$$

Now multiplying eq.(B5) from left by $(6a + 2)^{-1}((2a + 1)\not{p} - m)$, substracting it from eq.(B6) and using the definitions (B1) we have

$$-\frac{c\,m(3a + 1)}{8}\left((5m^2 + 3p^2)\,\xi + \frac{3\,c\,m}{B(a, b, c)}(\not{p} - 3m)\,\mathrm{g}^{\rho\sigma}\psi_{\rho\sigma}\right) = 0. \tag{B7}$$

From eq. (4) eq. (B7) we obtain

$$(3a + 1)\,c\,m^3\xi = 0. \tag{B8}$$

which means that the auxiliary field is vanishes provided $a \neq -\frac{1}{3}$. Having $\xi = 0$ the remaing constraints

$$\begin{aligned}
(\gamma_\mu p_\nu + \gamma_\nu p_\mu)\,\psi^{\mu\nu} &= 0, \\
p_\mu p_\nu\,\psi^{\mu\nu} &= 0, \\
\mathrm{g}_{\mu\nu}\,\psi^{\mu\nu} &= 0,
\end{aligned} \tag{B9}$$

can be easily derived from eqs.(B2-B4) and eq.(4).

Now multiplying eq. (3) from left by γ^ν and p^ν and using eqs. (B9) we have two equations

$$((a + 6F_1(a) - 1)\,\not{p} + (6F_2(a) - 1)m)(\gamma^\sigma \mathrm{g}^{\mu\rho} + \gamma^\rho \mathrm{g}^{\mu\sigma})\psi_{\rho\sigma} + 2(3a + 1)(\mathrm{g}^{\mu\rho}p^\sigma + p^\rho \mathrm{g}^{\mu\sigma})\psi_{\rho\sigma} = 0, \tag{B10}$$

$$((a + 1)\not{p} - m)(p^\sigma \mathrm{g}^{\mu\rho} + p^\rho \mathrm{g}^{\mu\sigma})\psi_{\rho\sigma} + (p^2(a + F_1(a)) + m\not{p}F_2(a)))(\gamma^\sigma \mathrm{g}^{\mu\rho} + \gamma^\rho \mathrm{g}^{\mu\sigma})\psi_{\rho\sigma} = 0. \tag{B11}$$

Again, multiplying eq. (B10) by $(6a + 2)^{-1}((a + 1)\not{p} - m)$, substracting the resulting equation from eq. (B11) and using definitions (B1) we get

$$(\gamma^\sigma \mathrm{g}^{\mu\rho} + \gamma^\rho \mathrm{g}^{\mu\sigma})\,\psi_{\rho\sigma} = 0, \tag{B12}$$

provided $a \neq -\frac{1}{3}, -\frac{1}{2}$. Then the constraint

$$(p^\sigma \mathrm{g}^{\mu\rho} + \gamma^\rho \mathrm{g}^{\mu\sigma})\,\psi_{\rho\sigma} = 0 \tag{B13}$$

immediately follows from eqs. (B10, B12). Having $\gamma^\rho\psi_{\rho\sigma} = 0$, $p^\rho\psi_{\rho\sigma} = 0$, $\xi = 0$, and $\mathrm{g}^{\rho\sigma}\psi_{\rho\sigma} = 0$ the equation (3) reduces to the Dirac equation $(\not{p} - m)\psi_{\rho\sigma} = 0$. Finally we have shown that all Rarita-Schwinger constraint can be obtained from eqs. (3, 4). Hence, the function $\psi_{\rho\sigma}$ obeying these equations describes the field with spin-$\frac{5}{2}$.

APPENDIX C: SPIN-$\frac{5}{2}$ PROPAGATOR

The solution of the equation 19 can be written in form

$$\begin{aligned}
G^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(p) &= \frac{1}{p^2 - m^2}\left((\not{p} + m)\,\mathcal{P}^{\frac{5}{2}}_{\mu\nu;\rho\sigma}(p) + \frac{p^2 - m^2}{m^2}\left(\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p) + \mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p)\right)\right), \\
G^{\rho\sigma}_{(\psi\xi)}(p) &= \frac{1}{64\,m^3c}\left((\not{p} + m)(2(\gamma^\rho p^\sigma + \gamma^\sigma p^\rho) + 5m\mathrm{g}^{\rho\sigma}) + 6(p^2 - m\not{p})\mathrm{g}^{\rho\sigma} - 16p^\rho p^\sigma\right),
\end{aligned} \tag{C1}$$

where the lower spin-$\frac{3}{2}$, -$\frac{1}{2}$ parts $\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p)$ and $\mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p)$ are

$$\mathcal{D}^{\frac{3}{2}}_{\mu\nu;\rho\sigma}(p) = -\frac{4}{5}(\not{p}+m)\mathcal{P}^{\frac{3}{2}}_{11;\,\mu\nu;\rho\sigma}(p) + \frac{m}{\sqrt{5}}\left(\mathcal{P}^{\frac{3}{2}}_{12;\,\mu\nu;\rho\sigma}(p) + \mathcal{P}^{\frac{3}{2}}_{21;\,\mu\nu;\rho\sigma}(p)\right) \tag{C2}$$

$$\begin{aligned}
\mathcal{D}^{\frac{1}{2}}_{\mu\nu;\rho\sigma}(p) = \frac{1}{80\,m^2}\Bigg(&-\frac{3}{8}\Big[\left((73m^2-12p^2)\not{p} + 3m(27m^2-8p^2)\right)\mathcal{P}^{\frac{1}{2}}_{11;\,\mu\nu;\rho\sigma}(p) \\
&+ \left((35m^2-36p^2)\not{p} - m(13m^2+96p^2)\right)\mathcal{P}^{\frac{1}{2}}_{22;\,\mu\nu;\rho\sigma}(p) \\
&-\sqrt{3}\left((43m^2-12p^2)\not{p} + m(47m^2-28p^2)\right)\left(\mathcal{P}^{\frac{1}{2}}_{12;\,\mu\nu;\rho\sigma}(p) + \mathcal{P}^{\frac{1}{2}}_{21;\,\mu\nu;\rho\sigma}(p)\right)\Big] \\
&- \left((16m^2+3p^2)\not{p} + m(16m^2-15p^2)\right)\mathcal{P}^{\frac{1}{2}}_{33;\,\mu\nu;\rho\sigma}(p) \\
&+\frac{9}{2\sqrt{6}}(3m^2-2p^2)\,\not{p}\,\left(\mathcal{P}^{\frac{1}{2}}_{13;\,\mu\nu;\rho\sigma}(p) - \mathcal{P}^{\frac{1}{2}}_{31;\,\mu\nu;\rho\sigma}(p)\right) \\
&+\frac{3m}{2\sqrt{6}}(64m^2-21p^2)\left(\mathcal{P}^{\frac{1}{2}}_{13;\,\mu\nu;\rho\sigma}(p) + \mathcal{P}^{\frac{1}{2}}_{31;\,\mu\nu;\rho\sigma}(p)\right) \\
&+\frac{9}{2\sqrt{2}}(3m^2+2p^2)\,\not{p}\,\left(\mathcal{P}^{\frac{1}{2}}_{23;\,\mu\nu;\rho\sigma}(p) - \mathcal{P}^{\frac{1}{2}}_{32;\,\mu\nu;\rho\sigma}(p)\right) \\
&-\frac{m}{2\sqrt{2}}(64m^2-69p^2)\left(\mathcal{P}^{\frac{1}{2}}_{23;\,\mu\nu;\rho\sigma}(p) + \mathcal{P}^{\frac{1}{2}}_{32;\,\mu\nu;\rho\sigma}(p)\right)\Bigg).
\end{aligned} \tag{C3}$$

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